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A Dirac particle in a time-varying magnetic field

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Abstract. A Dirac particle interaction with a uniform magnetic field $\boldsymbol{B}$ suddenly (by leaps) changing in time is considered theoretically. Characteristic features of the switching-on and switching-off processes are derived; in particular, it is shown that after switchings there is no spin flip if the component of the kinetic momentum along the field is equal to zero. Generalizations to more than one switching are presented. Various results of calculations of the switching process from $B_1\neq 0$ to $B_2\neq 0$ are also presented. Limiting non-relativistic and ultrarelativistic cases are discussed. In the non-relativistic case, derived equations transform to earlier known results. Difficulties of the one-particle interpretation are shown. Some new results of the non-relativistic case are given in an appendix.

In [1] a non-relativistic spinless particle in a magnetic field suddenly (by leaps) changing in time was considered theoretically. Direct matching of the wavefunctions before and after switchings was utilized for deriving characteristic features of such processes†. Here we want to extend this investigation to a relativistic case; namely, we will solve a Dirac equation with a magnetic field suddenly changing in time. Exact solutions of the one-particle Dirac equation in miscellaneous configurations are of great theoretical interest, and various approaches to these problems have been proposed, one of the most promising being the algebraic method of separation of variables proposed in [3] and developed in [4]. As was stressed in [1], in our arrangement at the times of switching, an infinitely strong electric field is induced, and, therefore, the processes of creation and destruction of particles play important roles. However, we will neglect these phenomena and will confine our considerations to the one-particle theory only. Also, as will be shown below, for very strong magnetic fields the Dirac equation does not hold. This is due to the wavepacket localization in strong fields in a region of space comparable with the Compton wavelength.

Let us start by considering the situation when a magnetic field $\boldsymbol{B}=(0,0,B)$ is suddenly switched on at the time $t=0$: $\boldsymbol{B}(t)=\boldsymbol{B}h(t)$, where $h(t)$ is the Heaviside step function. We choose the vector potential in the following Landau gauge: $\boldsymbol{A}=(0,Bx,0)$. At $t<0$ the particle is described by the wavefunction

$$\Psi(x,y,z,t)=\frac{1}{(2\pi\hbar)^{3/2}}\exp\left(-\mathrm{i}\frac{R}{\hbar}t\right)\exp\left(\frac{\mathrm{i}}{\hbar}(p_x(x-x_0)+p_yy+p_zz)\right)\varphi_s$$
$$(s=-1\text{ or }+1) \qquad (1)$$

† See also [2] where, among others, one particular result of [1] was obtained for a harmonic oscillator using the theory of explicitly time-dependent invariants.

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$$\varphi_{+1}=\frac{1}{\sqrt{2R(R+m_0c^2)}}\begin{pmatrix} R+m_0c^2 \\ 0 \\ cp_z \\ c(p_x+\mathrm{i}p_y) \end{pmatrix} \tag{2a}$$

$$\varphi_{-1}=\frac{1}{\sqrt{2R(R+m_0c^2)}}\begin{pmatrix} 0 \\ R+m_0c^2 \\ c(p_x-\mathrm{i}p_y) \\ -cp_z \end{pmatrix} \tag{2b}$$

where $R\equiv R(p_x, p_y, p_z)=+\sqrt{m_0^2c^4+c^2p_x^2+c^2p_y^2+c^2p_z^2}$, $x_0=p_y/eB$ and m_0 is the rest mass of the particle. It is easy to check that bispinors φ_s satisfy the orthonormality condition:

$$\varphi_s^+\varphi_{s'}=\delta_{ss'}.$$

After switching, one gets superposition of the relativistic magnetic states [5]:

$$\Psi(x, y, z, t)=\frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_y y+p_z z)\right)\sum_{n=0}^{\infty}\sum_{\nu=\pm1}C_{s\nu n}\chi_{\nu n}^{(M)}(x)\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\nu n}t\right) \tag{3}$$

$$\chi_{+1n}^{(M)}(x)=\frac{1}{\sqrt{2\varepsilon_{+1n}(\varepsilon_{+1n}+m_0c^2)}}\begin{pmatrix} (\varepsilon_{+1n}+m_0c^2)v_n(r_B, (x-x_0)/r_B) \\ 0 \\ cp_z v_n(r_B, (x-x_0)/r_B) \\ -\mathrm{i}\sqrt{2nm_0c^2\hbar\omega_B}\,v_{n-1}(r_B, (x-x_0)/r_B) \end{pmatrix} \tag{4a}$$

$$\chi_{-1n}^{(M)}(x)=\frac{1}{\sqrt{2\varepsilon_{-1n}(\varepsilon_{-1n}+m_0c^2)}}\begin{pmatrix} 0 \\ (\varepsilon_{-1n}+m_0c^2)v_n(r_B, (x-x_0)/r_B) \\ \mathrm{i}\sqrt{2(n+1)m_0c^2\hbar\omega_B}\,v_{n+1}(r_B, (x-x_0)/r_B) \\ -cp_z v_n(r_B, (x-x_0)/r_B) \end{pmatrix} \tag{4b}$$

$$\varepsilon_{\nu n}=+\sqrt{m_0^2c^4+c^2p_z^2+m_0c^2\hbar\omega_B(2n+1-\nu)} \tag{5}$$

$$v_n(r_B, \xi)=\frac{1}{\pi^{1/4}r_B^{1/2}\sqrt{2^n n!}}\exp\left(-\frac{\xi^2}{2}\right)H_n(\xi) \tag{6}$$

where $r_B=\sqrt{\hbar/eB}$, $\omega_B=eB/m_0$, and $H_n(\xi)$ are Hermite polynomials.

The functions $\chi_{\nu n}^{(M)}$ satisfy the equation

$$\int_{-\infty}^{\infty}\chi_{\nu n}^{(M)+}\chi_{\nu' n'}^{(M)}\,\mathrm{d}x=\delta_{\nu\nu'}\delta_{nn'}. \tag{7}$$

The factors $|C_{s\nu n}|^2$ are probabilities of a finding particle in the magnetic state $|n, \nu\rangle$, and

$$\sum_{n=0}^{\infty}\sum_{\nu=\pm1}|C_{s\nu n}|^2=1 \qquad s=\pm1. \tag{8}$$

Similar to [1], matching the wavefunction at $t=0$, one obtains an expression for C_{svn}:

$$C_{svn}=\int_{-\infty}^{\infty}\chi_{vn}^{(M)+}\varphi_s\exp\left(\frac{i}{\hbar}p_x(x-x_0)\right)dx. \tag{9}$$

Four possible cases of s and v are given below:

(i) $s=+1,\ v=+1$,

$$C_{+1,+1,n}=\frac{i^n}{2\sqrt{R(R+m_0c^2)\varepsilon_{+1n}(\varepsilon_{+1n}+m_0c^2)}}\times[(R+m_0c^2)(\varepsilon_{+1n}+m_0c^2)\beta_n+c^2p_z^2\beta_n+\Delta_n cp_x\beta_{n-1}+i\Delta_n cp_y\beta_{n-1}] \tag{10a}$$

(ii) $s=+1,\ v=-1$,

$$C_{+1,-1,n}=\frac{i^n cp_z}{2\sqrt{R(R+m_0c^2)\varepsilon_{-1n}(\varepsilon_{-1n}+m_0c^2)}}(\Delta_{n+1}\beta_{n+1}+cp_x\beta_n-icp_y\beta_n) \tag{10b}$$

(iii) $s=-1,\ v=-1$,

$$C_{-1,-1,n}=\frac{i^n}{2\sqrt{R(R+m_0c^2)\varepsilon_{-1n}(\varepsilon_{-1n}+m_0c^2)}}[(R+m_0c^2)(\varepsilon_{-1n}+m_0c^2)\beta_n+c^2p_z^2\beta_n+\Delta_{n+1}cp_x\beta_{n+1}+i\Delta_{n+1}cp_y\beta_{n+1}] \tag{10c}$$

(iv) $s=-1,\ v=+1$,

$$C_{-1,+1,n}=\frac{i^n cp_z}{2\sqrt{R(R+m_0c^2)\varepsilon_{+1n}(\varepsilon_{+1n}+m_0c^2)}}(\Delta_n\beta_{n-1}+cp_x\beta_n-icp_y\beta_n) \tag{10d}$$

with

$$\beta_n\equiv\beta_n\left(\frac{r_B}{\hbar}p_x\right)=\left(\frac{r_B}{\hbar}\right)^{1/2}\frac{1}{\pi^{1/4}\sqrt{2^n n!}}\exp\left[-\frac{1}{2}\left(\frac{r_B}{\hbar}p_x\right)^2\right]H_n\left(\frac{r_B}{\hbar}p_x\right) \tag{11}$$

$$\Delta_n=\sqrt{2nm_0c^2\hbar\omega_B}. \tag{12}$$

It is immediately seen from equations (10) that there is no spin flip if the component of the kinetic momentum along the field is equal to zero:

$$C_{ss'n}(p_z=0)=0 \qquad s\neq s'. \tag{13}$$

It follows from equations (10*b*) and (10*d*) that in the non-relativistic case there is also no spin flip. This justifies our neglect of spin in [1]. And, obviously, equations (10*a*) and (10*c*) transform in this case into equation (6) of [1].

Since the magnetic field acts perpendicularly to its direction, the most interesting case is when the y- and z-components of the kinetic momentum are zero. In this case

$$|C_{ssn}(p_z=p_y=0)|^2=\frac{1}{4}\left[\sqrt{\left(1+\frac{m_0c^2}{R}\right)\left(1+\frac{m_0c^2}{\varepsilon_{sn}}\right)}\beta_n+\sqrt{\left(1-\frac{m_0c^2}{R}\right)\left(1-\frac{m_0c^2}{\varepsilon_{sn}}\right)}\beta_{n-s}\right]^2$$

$$s=\pm 1. \tag{14}$$

Once again we see that in the non-relativistic case (ε_{vn}, $R \approx m_0c^2$), equation (14) transforms to the result obtained earlier [1]:

$$|C_{ssn}(p_z=p_y=0;\ \varepsilon_{vn},\ R\approx m_0c^2)|^2=\beta_n^2 \qquad s=\pm 1. \tag{15}$$

In the opposite ultrarelativistic case (ε_{vn}, $R \gg m_0c^2$) one gets

$$|C_{ssn}(p_z=p_y=0;\ \varepsilon_{vn},\ R\gg m_0c^2)|^2=\left(\frac{\beta_n+\beta_{n-s}}{2}\right)^2 \qquad s=\pm 1. \tag{16}$$

It is well known [6, 7] that at strong fields the one-particle interpretation of the Dirac equation faces enormous difficulties. These difficulties will be shown below.

It is easy to get a value for the average z-component of the spin $\langle\sigma_z\rangle_s$ after switching:

$$\langle\sigma_z\rangle_s=\frac{\hbar}{2}\sum_{n=0}^{\infty}\left[|C_{s,+1,n}|^2\left(1-\frac{\Delta_n^2}{\varepsilon_{+1n}(\varepsilon_{+1n}+m_0c^2)}\right)\right.$$
$$\left.-|C_{s,-1,n}|^2\left(1-\frac{\Delta_{n+1}^2}{\varepsilon_{-1n}(\varepsilon_{-1n}+m_0c^2)}\right)\right]. \tag{17}$$

It is seen that in the non-relativistic case

$$\langle\sigma_z\rangle_s=s(\hbar/2) \qquad s=\pm 1 \tag{18a}$$

as would be expected, and in the ultrarelativistic case

$$\langle\sigma_z\rangle_s=0. \tag{18b}$$

Let us now consider the solution of the problem when the magnetic field $\boldsymbol{B}$ is switched off at $t=0$: $\boldsymbol{B}(t)=\boldsymbol{B}h(-t)$. The wavefunctions are, at $t<0$,

$$\Psi=\frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_yy+p_zz)\right)\chi_{\mu m}^{(M)}\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\mu m}t\right) \tag{19}$$

and, at $t>0$,

$$\Psi=\frac{1}{(2\pi\hbar)^{3/2}}\exp\left(\frac{\mathrm{i}}{\hbar}(p_yy+p_zz)\right)$$
$$\times\sum_{s=\pm 1}\int_{-\infty}^{\infty}D_{s\mu m}(p_x)\exp\left(\frac{\mathrm{i}}{\hbar}p_x(x-x_0)\right)\exp\left(-\mathrm{i}\frac{R}{\hbar}t\right)\varphi_s\,\mathrm{d}p_x \tag{20}$$

$$D_{s\mu m}(p_x)=\frac{1}{(2\pi\hbar)^{1/2}}\int_{-\infty}^{\infty}\varphi_s^+\chi_{\mu m}^{(M)}\exp\left(-\frac{\mathrm{i}}{\hbar}p_x(x-x_0)\right)\mathrm{d}x. \tag{21}$$

$D_{s\mu m}(p_x)$ should obey the normalization condition:

$$\sum_{s=\pm 1}\int_{-\infty}^{\infty}|D_{s\mu m}(p_x)|^2\,\mathrm{d}p_x=1 \qquad \mu=\pm 1 \qquad m=0, 1, \ldots . \tag{22}$$

Comparison of equations (9) and (21) shows that

$$D_{svn}=C_{svn}^*. \tag{23}$$

In the non-relativistic case, the integration in equation (20) with $D_{s\mu m}$ given by equation (21) may be performed in the closed form (see the appendix). However, at relativistic speeds there is no analytical expression for integrals in equation (20) in the literature [8–12].

Now we want to discuss the difficulties of the one-particle interpretation mentioned above. In the ultrarelativistic case (and $p_z=0$)

$$|D_{ssm}|^2=\left(\frac{\beta_m+\beta_{m-s}}{2}\right)^2 \qquad s=\pm 1$$

with β_m given by (11). Using the properties of Hermite polynomials, one gets

$$\sum_{s'=\pm 1}\int_{-\infty}^{\infty}|D_{s'sm}(p_x)|^2\,\mathrm{d}p_x=\tfrac{1}{2}\neq 1. \tag{24}$$

Obviously, this is due to the negative energy states, which in the case $\varepsilon_{\nu n}, R\gg m_0c^2$ are of the same order as states with positive energies. Therefore, in the ultrarelativistic case after switchings one should take into account in equations (3) and (20) states with both positive (R, $\varepsilon_{\nu n}$) and negative ($-R$, $-\varepsilon_{\nu n}$) energies. Equations (8) and (22) which are exact at the non-relativistic speeds ($\varepsilon_{\nu n}, R\approx m_0c^2$), are violated in the ultrarelativistic case ($\varepsilon_{\nu n}, R\gg m_0c^2$). The larger the violation, the larger the probability of finding a particle in the state with negative energy. As is seen from equation (24), in the ultrarelativistic case the probability of finding a particle in the state with negative energy equals that for the states with positive energy. From a physical point of view, these transitions to negative energies are easily explained; namely, the electric field induced at the switching moments helps the particle to tunnel through the energy gap between states with different signs of the energy. These difficulties are similar to the Zitterbewegung or Klein paradox [6, 7].

In the same way, solutions are built for the case of more than one switching. For instance, if the magnetic field is switched on at $t=0$ and switched off at $t=T$ ($\boldsymbol{B}(t)=\boldsymbol{B}(h(t)-h(t-T))$), then the wavefunction at $t>T$ is†

$$\Psi=\frac{1}{(2\pi\hbar)^{3/2}}\exp\left(\frac{\mathrm{i}}{\hbar}(p_y y+p_z z)\right)\sum_{\mu=\pm 1}\int_{-\infty}^{\infty}C_{\mu s}(p_x,p_x')$$
$$\times\exp\left(\frac{\mathrm{i}}{\hbar}p_x'(x-x_0)\right)\exp\left(-\mathrm{i}\frac{R(p_x',p_y,p_z)}{\hbar}t\right)\varphi_\mu'\,\mathrm{d}p_x' \tag{25}$$

with φ_μ' given by equations (2) with replacement of p_x by p_x'. $C_{\mu s}(p_x,p_x')$ are expressed as

$$C_{\mu s}(p_x,p_x')=\exp\left(\mathrm{i}\frac{R(p_x',p_y,p_z)}{\hbar}T\right)\sum_{n=0}^{\infty}\sum_{\nu=\pm 1}C_{s\nu n}(p_x)C^*_{\mu\nu n}(p_x')\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\nu n}T\right) \tag{26}$$

and

$$|C_{\mu s}(p_x,p_x')|^2=\sum_{n,n'=0}^{\infty}\sum_{\nu,\nu'=\pm 1}C_{s\nu n}(p_x)C^*_{s\nu'n'}(p_x)C^*_{\mu\nu n}(p_x')C_{\mu\nu'n'}(p_x')$$
$$\times\exp\left(-\frac{\mathrm{i}}{\hbar}(\varepsilon_{\nu n}-\varepsilon_{\nu'n'})T\right). \tag{27}$$

† Obviously, after several switchings, transitions to the negative energy states are also possible. The probability of such a process for the uniform (non-space-dependent) vector potential A is described, for example, in [13].

If the magnetic field is switched off at $t=0$ and switched on again at $t=T$ ($B(t)=B(h(-t)+h(t-T))$), then the wavefunction at $t<0$ is expressed by equation (19), at $0<t<T$ by equation (20), and at $t>T$ by

$$\Psi(x,y,z,t)=\frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_y y+p_z z)\right)\sum_{n=0}^{\infty}\sum_{\nu=\pm1}C_{\nu n\mu m}\chi_{\nu n}^{(M)}\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\nu n}t\right) \tag{28}$$

with

$$\sum_{n=0}^{\infty}\sum_{\nu=\pm1}|C_{\nu n\mu m}|^2=1 \qquad \mu=\pm1 \qquad m=0,1,\ldots \tag{29}$$

$$C_{\nu n\mu m}=\exp\left(\frac{\mathrm{i}}{\hbar}\varepsilon_{\nu n}T\right)\sum_{s=\pm1}\int_{-\infty}^{\infty}C_{s\nu n}(p_x)C^{*}_{s\mu m}(p_x)\exp\left(-\mathrm{i}\frac{R}{\hbar}t\right)\mathrm{d}p_x. \tag{30}$$

In the non-relativistic case, equations (26), (27) and (30) transform to the corresponding equations (19), (20) and (23a) in [1]†. Comparison of equations (26) and (30) shows a simple way for constructing solutions for the case of more than two switchings.

The next problem we want to tackle is the process in which the magnetic field is suddenly changing from $B_1\neq0$ to $B_2\neq0$. In this case the initial wavefunction is

$$\Psi=\frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_y y+p_z z)\right)\chi_{\mu m}^{(1)}\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\mu m}^{(1)}t\right). \tag{31}$$

After switching ($t>0$) one gets

$$\Psi=\frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_y y+p_z z)\right)\sum_{n=0}^{\infty}\sum_{\nu=\pm1}C_{\nu n\mu m}\chi_{\nu n}^{(2)}\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\nu n}^{(2)}t\right). \tag{32}$$

Here

$$\chi_{+1n}^{(j)}(x)=\frac{1}{\sqrt{2\varepsilon_{+1n}^{(j)}(\varepsilon_{+1n}^{(j)}+m_0c^2)}}\begin{pmatrix}(\varepsilon_{+1n}^{(j)}+m_0c^2)v_n(r_j,(x-x_j)/r_j)\\ 0\\ cp_z v_n(r_j,(x-x_j)/r_j)\\ -\mathrm{i}\Delta_n^{(j)}v_{n-1}(r_j,(x-x_j)/r_j)\end{pmatrix} \tag{33a}$$

$$\chi_{-1n}^{(j)}(x)=\frac{1}{\sqrt{2\varepsilon_{-1n}^{(j)}(\varepsilon_{-1n}^{(j)}+m_0c^2)}}\begin{pmatrix}0\\ (\varepsilon_{-1n}^{(j)}+m_0c^2)v_n(r_j,(x-x_j)/r_j)\\ \mathrm{i}\Delta_{n+1}^{(j)}v_{n+1}(r_j,(x-x_j)/r_j)\\ -cp_z v_n(r_j,(x-x_j)/r_j)\end{pmatrix} \tag{33b}$$

$$\Delta_n^{(j)}=\sqrt{2nm_0c^2\hbar\omega_j} \qquad \varepsilon_{\nu n}^{(j)}=+\sqrt{m_0^2c^4+c^2p_z^2+m_0c^2\hbar\omega_j(2n+1-\nu)}$$

$$r_j=(\hbar/eB_j)^{1/2} \qquad x_j=p_y/eB_j \qquad \omega_j=eB_j/m_0 \qquad j=1,2.$$

† There are misprints in equations (19), (23a), (24b) and (36) in [1]; namely, terms in the first exponents in these equations should have positive signs.

Similar to the previous cases

$$\sum_{n=0}^{\infty} \sum_{\nu=\pm 1} |C_{\nu n \mu m}|^2 = 1 \qquad \mu = \pm 1 \qquad m = 0, 1, \ldots . \tag{34}$$

Matching solutions, one obtains

$$C_{\nu n \mu m}(B_2, B_1) = \int_{-\infty}^{\infty} \chi_{\nu n}^{(2)+} \chi_{\mu m}^{(1)} \, \mathrm{d}x. \tag{35}$$

For four cases we get:
(i) $\mu = +1, \nu = +1,$

$$C_{+1,n,+1,m} = \frac{1}{2\sqrt{\varepsilon_{+1n}^{(2)}(\varepsilon_{+1n}^{(2)} + m_0c^2)\varepsilon_{+1m}^{(1)}(\varepsilon_{+1m}^{(1)} + m_0c^2)}}$$
$$\times ((\varepsilon_{+1m}^{(1)} + m_0c^2)(\varepsilon_{+1n}^{(2)} + m_0c^2)\alpha_{nm} + c^2p_z^2\alpha_{nm} + \Delta_n^{(2)}\Delta_m^{(1)}\alpha_{n-1,m-1}) \tag{36a}$$

(ii) $\mu = +1, \nu = -1,$

$$C_{-1,n,+1,m} = \frac{\mathrm{i}cp_z}{2\sqrt{\varepsilon_{-1n}^{(2)}(\varepsilon_{-1n}^{(2)} + m_0c^2)\varepsilon_{+1m}^{(1)}(\varepsilon_{+1m}^{(1)} + m_0c^2)}}(\Delta_m^{(1)}\alpha_{n,m-1} - \Delta_{n+1}^{(2)}\alpha_{n+1,m}) \tag{36b}$$

(iii) $\mu = -1, \nu = -1,$

$$C_{-1,n,-1,m} = \frac{1}{2\sqrt{\varepsilon_{-1n}^{(2)}(\varepsilon_{-1n}^{(2)} + m_0c^2)\varepsilon_{-1m}^{(1)}(\varepsilon_{-1m}^{(1)} + m_0c^2)}}$$
$$\times ((\varepsilon_{-1m}^{(1)} + m_0c^2)(\varepsilon_{-1n}^{(2)} + m_0c^2)\alpha_{nm} + c^2p_z^2\alpha_{nm} + \Delta_{n+1}^{(2)}\Delta_{m+1}^{(1)}\alpha_{n+1,m+1}) \tag{36c}$$

(iv) $\mu = -1, \nu = +1,$

$$C_{+1,n,-1,m} = \frac{\mathrm{i}cp_z}{2\sqrt{\varepsilon_{+1n}^{(2)}(\varepsilon_{+1n}^{(2)} + m_0c^2)\varepsilon_{-1m}^{(1)}(\varepsilon_{-1m}^{(1)} + m_0c^2)}}$$
$$\times (\Delta_{m+1}^{(1)}\alpha_{n,m+1} - \Delta_n^{(2)}\alpha_{n-1,m}). \tag{36d}$$

$\alpha_{nm} \equiv \alpha_{nm}(B_2, B_1)$ are known from the non-relativistic case [1]:

$$\alpha_{nm}(B_2, B_1) = \int_{-\infty}^{\infty} v_n(r_2, (x - x_2)/r_2) v_m(r_1, (x - x_1)/r_1) \, \mathrm{d}x$$
$$= \exp\left(-\frac{1}{2}\frac{x_1}{r_1}\frac{x_2}{r_2}\frac{(B_2 - B_1)^2}{(B_1 + B_2)\sqrt{B_1B_2}}\right)\sqrt{\frac{\sqrt{B_1B_2}}{(B_1 + B_2)2^{n+m-1}n!m!}}$$
$$\times \left(\frac{B_1 - B_2}{B_1 + B_2}\right)^{m/2}\left(\frac{B_2 - B_1}{B_2 + B_1}\right)^{n/2} \sum_{k=0}^{\min(m,n)} \binom{m}{k}\binom{n}{k} k! \left(1 - \frac{B_1}{B_2}\right)^{k/2}$$
$$\times \left(1 - \frac{B_2}{B_1}\right)^{k/2} H_{m-k}\left(-\frac{x_1}{r_1}\sqrt{\frac{B_1 - B_2}{B_1 + B_2}}\right) H_{n-k}\left(-\frac{x_2}{r_2}\sqrt{\frac{B_2 - B_1}{B_2 + B_1}}\right). \tag{37}$$

Once again it is seen that there is no spin flip (i) at $p_z=0$ and (ii) in the non-relativistic case:

$$C_{\nu n\nu' m}(p_z=0)=0 \qquad \nu\neq\nu' \tag{38a}$$

$$C_{\nu n\nu' m}(\varepsilon_{\nu n},\, \varepsilon_{\nu' m}\approx m_0c^2)=0 \qquad \nu\neq\nu'. \tag{38b}$$

And, of course, in the non-relativistic case, equations (36*a*) and (36*c*) transform to the corresponding expression (31) in [1]:

$$C_{\nu n\nu' m}=\alpha_{nm}\delta_{\nu\nu'}. \tag{39}$$

At $p_z=0$ it follows from equations (35*a*) and (35*c*) that

$$|C_{\nu n\nu m}(p_z=0)|^2=\frac{1}{4}\left[\sqrt{\left(1+\frac{m_0c^2}{\varepsilon_{\nu m}^{(1)}}\right)\left(1+\frac{m_0c^2}{\varepsilon_{\nu n}^{(2)}}\right)}\alpha_{nm}\right.$$
$$\left.+\sqrt{\left(1-\frac{m_0c^2}{\varepsilon_{\nu m}^{(1)}}\right)\left(1-\frac{m_0c^2}{\varepsilon_{\nu n}^{(2)}}\right)}\alpha_{n-\nu,m-\nu}\right]^2. \tag{40}$$

A clear analogy with equation (14) is seen. Therefore, for the case $B_1\neq 0$, $B_2\neq 0$ all properties discussed above will be valid also; namely, in the non-relativistic case,

$$|C_{\nu n\nu m}|^2=\alpha_{nm}^2 \tag{40a}$$

and for the ultrarelativistic case

$$|C_{\nu n\nu m}|^2=\left(\frac{\alpha_{nm}+\alpha_{n-\nu,m-\nu}}{2}\right)^2. \tag{40b}$$

As a final example, we consider the situation where $B_1\equiv -B_2\equiv B$. The wavefunction at $t<0$ is expressed by equation (19), and at $t>0$ it is

$$\Psi(x,y,z,t)=\frac{1}{2\pi\hbar}\exp\left(\frac{\mathrm{i}}{\hbar}(p_yy+p_zz)\right)\sum_{n=0}^{\infty}\sum_{\nu=\pm 1}C_{\nu n\mu m}\chi_{\nu n}^{(+)}(x)\exp\left(-\frac{\mathrm{i}}{\hbar}\varepsilon_{\nu n}t\right) \tag{41}$$

with $\chi_{\nu n}^{(+)}(x)$ being expressed by equations (4) with the opposite sign of x_0. $C_{\nu n\mu m}$ are

$$C_{\nu n\mu m}=\int_{-\infty}^{\infty}\chi_{\nu n}^{(+)+}\chi_{\mu m}^{(M)}\,\mathrm{d}x. \tag{42}$$

(i) $\mu=+1$, $\nu=+1$,

$$C_{+1,n,+1,m}=\frac{1}{2\sqrt{\varepsilon_{+1n}(\varepsilon_{+1n}+m_0c^2)\varepsilon_{+1m}(\varepsilon_{+1m}+m_0c^2)}}$$
$$\times((\varepsilon_{+1m}+m_0c^2)(\varepsilon_{+1n}+m_0c^2)\Gamma_{nm}+c^2p_z^2\Gamma_{nm}+\Delta_n\Delta_m\Gamma_{n-1,m-1}) \tag{42a}$$

(ii) $\mu=+1$, $\nu=-1$,

$$C_{-1,n,+1,m}=\frac{\mathrm{i}cp_z}{2\sqrt{\varepsilon_{-1n}(\varepsilon_{-1n}+m_0c^2)\varepsilon_{+1m}(\varepsilon_{+1m}+m_0c^2)}}(\Delta_m\Gamma_{n,m-1}-\Delta_{n+1}\Gamma_{n+1,m}) \tag{42b}$$

(iii) $\mu=-1$, $\nu=-1$,

$$C_{-1,n,-1,m}=\frac{1}{2\sqrt{\varepsilon_{-1n}(\varepsilon_{-1n}+m_0c^2)\varepsilon_{-1m}(\varepsilon_{-1m}+m_0c^2)}}\times((\varepsilon_{-1m}+m_0c^2)(\varepsilon_{-1n}+m_0c^2)\Gamma_{nm}+c^2p_z^2\Gamma_{nm}+\Delta_{n+1}\Delta_{m+1}\Gamma_{n+1,m+1}) \quad (42c)$$

(iv) $\mu=-1$, $\nu=+1$,

$$C_{+1,n,-1,m}=\frac{icp_z}{2\sqrt{\varepsilon_{+1n}(\varepsilon_{+1n}+m_0c^2)\varepsilon_{-1m}(\varepsilon_{-1m}+m_0c^2)}}(\Delta_{m+1}\Gamma_{n,m+1}-\Delta_n\Gamma_{n-1,m}) \quad (42d)$$

and [1]

$$\Gamma_{nm}=\int_{-\infty}^{\infty} v_n(r_B,(x+x_0)/r_B)v_m(r_B,(x-x_0)/r_B)\,dx$$

$$=\exp\left(-\frac{x_0^2}{r_B^2}\right)\begin{cases}\sqrt{2^{n-m}\dfrac{m!}{n!}}\left(-\dfrac{x_0}{r_B}\right)^{n-m}L_m^{n-m}\left(2\dfrac{x_0^2}{r_B^2}\right) & n\geqslant m\\ \sqrt{2^{m-n}\dfrac{n!}{m!}}\left(\dfrac{x_0}{r_B}\right)^{m-n}L_n^{m-n}\left(2\dfrac{x_0^2}{r_B^2}\right) & m>n.\end{cases} \quad (43)$$

$L_n^m(\xi)$ are Laguerre polynomials.

From the properties of Γ_{nm} [1], it follows that at $p_y=0$ there are no transitions:

$$C_{\nu n\mu m}(p_y=0)=\delta_{\nu\mu}\delta_{nm}. \quad (44)$$

This is explained by the fact that although relativistic considerations taking into account spin effects remove the degeneracy of the magnetic levels with respect to the field direction, some degeneracy remains: states $|n,\uparrow\rangle$ and $|n-1,\downarrow\rangle$ have the same energy (see equation (5)).

Expressions for several switchings from one non-zero field to another are easily derived analogously to the procedure discussed above and in [1].

In conclusion, the non-relativistic treatment of [1] has been extended in the present paper to a theoretical investigation of a Dirac particle interaction with a magnetic field suddenly changing in time; its characteristic features have been defined and the drawbacks of the one-particle interpretation pointed out. The next obvious step is to overcome these difficulties. However, this is a subject for special consideration.

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Appendix

We want to discuss here one interesting result of the switching-off procedure in the non-relativistic case which escaped our attention in [1]. If R, $\varepsilon_{\mu m}\approx m_0c^2$, then equations

(20) and (21) transform to equations (A1) and (A2), respectively (we write the dependence on the variables x and t only),

$$\Psi(x,t)=\frac{1}{(2\pi\hbar)^{1/2}}\int_{-\infty}^{\infty}D_m(p_x)\exp\left(\frac{\mathrm{i}}{\hbar}p_x(x-x_0)\right)\exp\left(-\mathrm{i}\frac{p_x^2}{2m_0\hbar}t\right)\mathrm{d}p_x \tag{A1}$$

$$D_m(p_x)=(-\mathrm{i})^m\beta_m\left(\frac{r_B}{\hbar}p_x\right) \tag{A2}$$

which are similar to equations (15) and (16) of [1].

Substituting equation (A2) into equation (A1) and calculating the integral [9, 11, 12]†, one gets

$$\Psi(x,t)=\frac{\mathrm{i}^m}{\pi^{1/4}(2^m m!)^{1/2}}\frac{1}{r_B^{1/2}\sqrt{1+\mathrm{i}\omega_B t}}\left(\frac{-1+\mathrm{i}\omega_B t}{1+\mathrm{i}\omega_B t}\right)^{m/2}$$
$$\times\exp\left(-\frac{(x-x_0)^2}{2r_B^2(1+i\omega_B t)}\right)H_m\left(\frac{x-x_0}{r_B\sqrt{1+\omega_B^2t^2}}\right) \tag{A3}$$

and

$$|\Psi(x,t)|^2=\frac{1}{\pi^{1/2}2^m m!}\frac{1}{r_B\sqrt{1+\omega_B^2t^2}}\exp\left(-\frac{(x-x_0)^2}{r_B^2(1+\omega_B^2t^2)}\right)H_m^2\left(\frac{x-x_0}{r_B\sqrt{1+\omega_B^2t^2}}\right)$$
$$\equiv v_m^2(r_{\mathrm{eff}},(x-x_0)/r_{\mathrm{eff}})). \tag{A4}$$

Therefore, we can stress that after switching off the magnetic field the particle remains in the state with the same number m. The centre of magnetic oscillations x_0 is also conserved. The magnetic radius, however, is now time-dependent (for convenience, below we have denoted the values of the magnetic field, magnetic radius and cyclotron frequency before switching as B_0, r_0 and ω_0, respectively):

$$r_{\mathrm{eff}}(t)=r_0\sqrt{1+\omega_0^2t^2}. \tag{A5}$$

It follows from equation (A5) that the 'instantaneous' value of the 'effective' magnetic field is

$$B^{\mathrm{eff}}(t)=B_0/(1+\omega_0^2t^2). \tag{A6}$$

In some sense, we can say that although the magnetic field B_0 suddenly vanishes at $t=0$, the parabolic potential well formed by it, does not disappear, but becomes more and more gently sloping, according to equation (A6), with the centre of the well being unchanged. This in a natural way explains the fact that after the subsequent switching on of the field at $t=T$, transitions are possible only between states with the same parity [1].

Thus, after the sudden switching off of the magnetic field at $t=0$ one can calculate the probability of finding a particle at the point x at time $t>0$, using the usual expressions for the wavefunctions v_m in a uniform magnetic field (equation (6)) with the 'effective' magnetic field and magnetic radius given by equations (A6) and (A5).

† We want to point out a further two errors in [9]; namely, equations (2.20.3.12) and (2.20.5.2) are wrong. The correct forms of these integrals may be found from the corresponding equations in [11, 12].

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